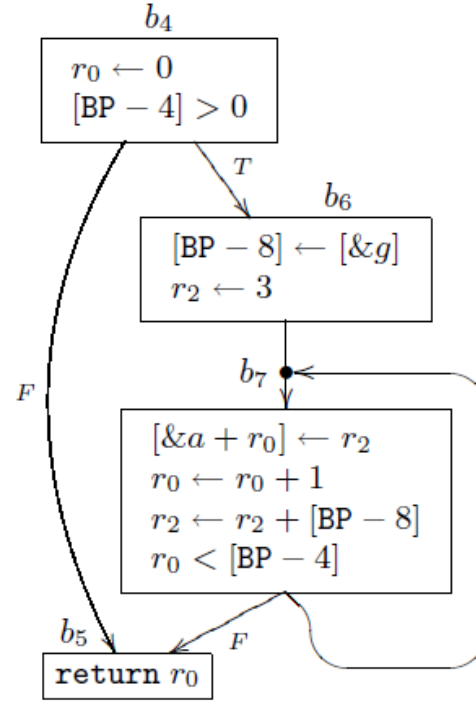


(a) Original program



(b) Transformed program

### Transfer functions

$$b_0(m, n) = b_1(m, 0, n)$$

$$b_1(m, i, n) = (i < n) ? b_3(m, i, n) : b_2(m, i)$$

$$b_2(m, i) = \text{ret}(m, i)$$

$$b_3(m, i, n) = b_1(\text{upd}(m, \&a + i, (\text{sel}(m, \&g) * i + 3)), i + 1, n)$$

$$b_4(m, [\text{BP} - 4]) = ([\text{BP} - 4] > 0) ? b_6(m, [\text{BP} - 4], 0) : b_5(m, 0)$$

$$b_5(m, r_0) = \text{ret}(m, r_0)$$

$$b_6(m, [\text{BP} - 4], r_0) = b_7(m, [\text{BP} - 4], r_0, [\text{BP} - 8], 3)$$

$$b_7(m, [\text{BP} - 4], [\text{BP} - 8], r_0, r_2) = (r_0 + 1 < [\text{BP} - 4]) ? b_7(\text{upd}(m, \&a + r_0, r_2), r_0 + 1, r_2 + [\text{BP} - 8]) : b_5(\text{upd}(m, \&a + r_0, r_2), r_0)$$

### Infer

Done = 0

Todo =  $b_0, b_4$

Done =  $b_0, b_4$

Todo = 0

Scan( $b_0(m^{0.4}, n^{0.4}), \square, b_4(m^{0.4}, [\text{BP} - 4]^{0.4}), \square$ )

Scan( $b_1(m^{0.4}, 0, n^{0.4}), \square, (i^{1.4} = 0), b_4(m^{0.1}, [\text{BP} - 4]^{0.4})$ )

$C = C \cup \{0 < n^{0.4} \equiv [\text{BP} - 4]^{0.4} > 0\}$

MarkRelated( $b_3(m^{0.4}, 0, n^{0.4}), b_6(m^{0.4}, [\text{BP} - 4]^{0.4}, 0)$ )

$C = C \cup (b_3(m^{0.4}, 0, n^{0.4}) = b_6(m^{0.4}, [\text{BP} - 4]^{0.4}, 0))$

Todo +=  $b_3, b_6$

MarkRelated( $b_2(m^{0.4}, 0), b_5(m^{0.4}, 0)$ )

$C = C \cup (b_2(m^{0.4}, 0) = b_5(m^{0.4}, 0))$

Todo +=  $b_2, b_5$

Done = ( $b_0, b_4$ )

Todo = ( $b_2, b_5$ ), ( $b_3, b_6$ )

Done = ( $b_0, b_4$ ), ( $b_2, b_5$ )

Todo = ( $b_3, b_6$ )

Scan( $b_2(m^{2.5}, i^{2.5}), \square, b_5(m^{2.5}, r_0^{2.5}), \square$ )

$C = C \cup (i^{2.5} = r_0^{2.5})$

Done = (b<sub>0</sub>, b<sub>4</sub>), (b<sub>2</sub>, b<sub>5</sub>)

Todo = (b<sub>3</sub>, b<sub>6</sub>)

Done = (b<sub>0</sub>, b<sub>4</sub>), (b<sub>2</sub>, b<sub>5</sub>), (b<sub>3</sub>, b<sub>6</sub>)

Todo = 0

Scan(b<sub>3</sub>(m<sup>3,6</sup>, i<sup>3,6</sup>, n<sup>3,6</sup>), □, b<sub>6</sub>(m<sup>3,6</sup>, r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>), □)

Scan(b<sub>1</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup> + 1, n<sup>3,6</sup>), □, b<sub>6</sub>(m<sup>3,6</sup>, r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>), □)

Scan(b<sub>1</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup> + 1, n<sup>3,6</sup>), □, b<sub>7</sub>(m<sup>3,6</sup>, r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>, [BP-8], 3), □)

C = C U {(i<sup>3,6</sup>+1 < n<sup>3,6</sup> = r<sub>0</sub><sup>3,6</sup>+1 < [BP-4]<sup>3,6</sup>)}

MarkRelated(b<sub>3</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>, n<sup>3,6</sup>), b<sub>7</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>, [BP-8], 3))

C = C U (b<sub>3</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>, n<sup>3,6</sup>) = b<sub>7</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>, [BP-8], 3))

Todo += b<sub>3</sub>, b<sub>7</sub>

MarkRelated(b<sub>2</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>), b<sub>5</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>))

C = C U (b<sub>2</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>) = b<sub>5</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>))

Done = (b<sub>0</sub>, b<sub>4</sub>), (b<sub>2</sub>, b<sub>5</sub>), (b<sub>3</sub>, b<sub>6</sub>)

Todo = b<sub>3</sub>, b<sub>7</sub>

Done = (b<sub>0</sub>, b<sub>4</sub>), (b<sub>2</sub>, b<sub>5</sub>), (b<sub>3</sub>, b<sub>6</sub>), (b<sub>3</sub>, b<sub>7</sub>)

Todo = 0

Scan(b<sub>3</sub>(m<sup>3,7</sup>, i<sup>3,7</sup>, n<sup>3,7</sup>), □, b<sub>7</sub>(m<sup>3,7</sup>, [BP - 4]<sup>3,7</sup>, [BP - 8]<sup>3,7</sup>, r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), □)

Scan(b<sub>1</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup> + 1, n<sup>3,7</sup>), □, b<sub>7</sub>(m<sup>3,7</sup>, [BP - 4]<sup>3,7</sup>, [BP - 8]<sup>3,7</sup>, r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), □)

C = C U (i<sup>3,7</sup> + 1 < n<sup>3,7</sup> ≡ r<sub>0</sub><sup>3,7</sup> + 1 < [BP - 4]<sup>3,7</sup>)

MarkRelated(b<sub>3</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup>+1, n<sup>3,7</sup>), b<sub>7</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1, r<sub>2</sub><sup>3,7</sup> + [BP - 8]<sup>3,7</sup>))

C = C U (b<sub>3</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup> + 1, n<sup>3,7</sup>) = b<sub>7</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1, r<sub>2</sub><sup>3,7</sup> + [BP - 8]<sup>3,7</sup>))

MarkRelated(b<sub>2</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup>+1), b<sub>5</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1))

C = C U (b<sub>2</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup>+1) = b<sub>5</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1))

Done = (b<sub>0</sub>, b<sub>4</sub>), (b<sub>2</sub>, b<sub>5</sub>), (b<sub>3</sub>, b<sub>6</sub>), (b<sub>3</sub>, b<sub>7</sub>)

Todo = 0

*done!*

## Constraints

1. [BP - 4] = n
2. [BP - 8] = &g
3. 0 < n<sup>0,4</sup> ≡ [BP - 4]<sup>0,4</sup> > 0
4. b<sub>3</sub>(m<sup>0,4</sup>, 0, n<sup>0,4</sup>) = b<sub>6</sub>(m<sup>0,4</sup>, [BP - 4]<sup>0,4</sup>, 0)
5. b<sub>2</sub>(m<sup>0,4</sup>, 0) = b<sub>5</sub>(m<sup>0,4</sup>, 0)
6. i<sup>2,5</sup> = r<sub>0</sub><sup>2,5</sup>
7. i<sup>3,6</sup>+1 < n<sup>3,6</sup> = r<sub>0</sub><sup>3,6</sup>+1 < [BP-4]<sup>3,6</sup>
8. b<sub>3</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>, n<sup>3,6</sup>) = b<sub>7</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>, [BP - 4]<sup>3,6</sup>, [BP-8], 3)
9. b<sub>2</sub>(upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3), i<sup>3,6</sup>) = b<sub>5</sub>(upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3), r<sub>0</sub><sup>3,6</sup>)
10. i<sup>3,6</sup> = r<sub>0</sub><sup>3,6</sup>
11. upd(m<sup>3,6</sup>, &a+i<sup>3,6</sup>, (sel(m<sup>3,6</sup>, &g<sup>3,6</sup>) \* i<sup>3,6</sup> + 3) = upd(m<sup>3,6</sup>, a+r<sub>0</sub><sup>3,6</sup>, 3)
12. i<sup>3,7</sup> + 1 < n<sup>3,7</sup> ≡ r<sub>0</sub><sup>3,7</sup> + 1 < [BP - 4]<sup>3,7</sup>
13. b<sub>3</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup> + 1, n<sup>3,7</sup>) = b<sub>7</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1, r<sub>2</sub><sup>3,7</sup> + [BP - 8]<sup>3,7</sup>)
14. b<sub>2</sub>(upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3), i<sup>3,7</sup>+1) = b<sub>5</sub>(upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>), r<sub>0</sub><sup>3,7</sup> + 1)
15. i<sup>3,7</sup>+1 = r<sub>0</sub><sup>3,7</sup>+1 => i<sup>3,7</sup> = r<sub>0</sub><sup>3,7</sup>
16. upd(m<sup>3,7</sup>, &a+i<sup>3,7</sup>, (sel(m<sup>3,7</sup>, &g<sup>3,7</sup>) \* i<sup>3,7</sup> + 3) = upd(m<sup>3,7</sup>, &a+r<sub>0</sub><sup>3,7</sup>, r<sub>2</sub><sup>3,7</sup>)

10 and 11 come from 9

15 and 16 come from 14

## Solve

Substitute [BP-4] to n and [BP - 8] to &g and  $r_0^{2.5}$  to  $i^{2.5}$ ,  $i^{3.6}$  to  $r_0^{3.6}$  &  $r_0^{3.7}$  to  $i^{3.7}$  in all equations except 1, 2, 6, 10 & 15 respt.

1. [BP - 4] = n
2. [BP - 8] = &g
3.  $0 < n^{0.4} \equiv n^{0.4} > 0$
4.  $b_3(m^{0.4}, 0, n^{0.4}) = b_6(m^{0.4}, n^{0.4}, 0)$
5.  $b_2(m^{0.4}, 0) = b_5(m^{0.4}, 0)$
6.  $i^{2.5} = r_0^{2.5}$
7.  $i^{3.6}+1 < n^{3.6} = i^{3.6}+1 < n^{3.6}$
8.  $b_3(\text{upd}(m^{3.6}, \&a+i^{3.6}, (\text{sel}(m^{3.6}, \&g^{3.6}) * i^{3.6} + 3), i^{3.6}, n^{3.6}), r_0^{3.6}, n^{3.6}, \&g, 3)$
9.  $b_2(\text{upd}(m^{3.6}, \&a+i^{3.6}, (\text{sel}(m^{3.6}, \&g^{3.6}) * i^{3.6} + 3), i^{3.6}), r_0^{3.6}, r_0^{3.6}, \&g, 3)$
10.  $i^{3.6} = r_0^{3.6}$
11.  $\text{upd}(m^{3.6}, \&a+i^{3.6}, (\text{sel}(m^{3.6}, \&g^{3.6}) * i^{3.6} + 3) = \text{upd}(m^{3.6}, \&a+i^{3.6}, 3)$
12.  $i^{3.7} + 1 < n^{3.7} \equiv i^{3.7} + 1 < n^{3.7}$
13.  $b_3(\text{upd}(m^{3.7}, \&a+i^{3.7}, (\text{sel}(m^{3.7}, \&g^{3.7}) * i^{3.7} + 3), i^{3.7} + 1, n^{3.7}), r_0^{3.7} + 1, r_2^{3.7} + \&g^{3.7})$
14.  $b_2(\text{upd}(m^{3.7}, \&a+i^{3.7}, (\text{sel}(m^{3.7}, \&g^{3.7}) * i^{3.7} + 3), i^{3.7}+1), r_0^{3.7} + 1, r_2^{3.7} + \&g^{3.7})$
15.  $i^{3.7}+1 = r_0^{3.7}+1 \Rightarrow i^{3.7} = r_0^{3.7}$
16.  $\text{upd}(m^{3.7}, \&a+i^{3.7}, (\text{sel}(m^{3.7}, \&g^{3.7}) * i^{3.7} + 3) = \text{upd}(m^{3.7}, \&a+i^{3.7}, r_2^{3.7})$

After substitution, most of them are now tautology.

Solving memory equivalence eq. 10 and 14

$$\text{upd}(m^{3.6}, \&a+i^{3.6}, (\text{sel}(m^{3.6}, \&g^{3.6}) * i^{3.6} + 3) = b_5(\text{upd}(m^{3.6}, \&a+i^{3.6}, 3)$$

$$\text{since } \&a+i^{3.6} == \&a+i^{3.6}$$

$$\text{Therefore } (\text{sel}(m^{3.6}, \&g^{3.6}) * i^{3.6} + 3 = 3$$

$$\Rightarrow i^{3.6} \text{ should be } 0$$

$$\text{upd}(m^{3.7}, \&a+i^{3.7}, (\text{sel}(m^{3.7}, \&g^{3.7}) * i^{3.7} + 3) = \text{upd}(m^{3.7}, \&a+i^{3.7}, r_2^{3.7})$$

$$(\text{sel}(m^{3.7}, \&g^{3.7}) * i^{3.7} + 3 = r_2^{3.7}$$

$$\text{That is: } [\&g^{3.7}] * i^{3.7} + 3 = r_2^{3.7}$$

## Simulation Relation

0	4	[BP - 4] = n	from 1
2	5	$i = r_0$	from 6
3	6	$i = r_0$ $i = 0$ [BP - 4] = n	from 8 from mem eq from 6 and above
3	7	$i = r_0$ [BP - 4] = n [BP - 8] = &g [&g] * i + 3 = $r_2$	from 15 from mem eqn